1.5 Complex Numbers

Definition of a Complex Number

If a and b are real numbers, the number $a + bi$ is a **complex number,** and it is said to be written in **standard form.** If $b = 0$, the number $a + bi = a$ is a real number. If $b \neq 0$, the number $a + bi$ is called an **imaginary number.** A number of the form bi, where $b \neq 0$, is called a **pure imaginary number.**

Equality of Complex Numbers

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other

$$a + bi = c + di$$ Equality of two complex numbers

if and only if $a = c$ and $b = d$.

In Exercises 1–4, find real numbers *a* and *b* such that the equation is true.

4. $(a + 6) + 2bi = 6 - 5i$

In Exercises 5–16, write the complex number in standard form.

6. $3 + \sqrt{-16}$

10. $\sqrt{-4}$

Addition and Subtraction of Complex Numbers

If $a + bi$ and $c + di$ are two complex numbers written in standard form, their sum and difference are defined as follows.

Sum: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference: $(a + bi) - (c + di) = (a - c) + (b - d)i$

The **additive identity** in the complex number system is zero (the same as in the real number system). Furthermore, the **additive inverse** of the complex number $a + bi$ is

$-(a + bi) = -a - bi.$ Additive inverse

So, you have

$(a + bi) + (-a - bi) = 0 + 0i = 0.$

In Exercises 17–26, perform the addition or subtraction and write the result in standard form.

22. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right)$

In Exercises 27–40, perform the operation and write the result in standard form.

30. $\left(\sqrt{-75}\right)^2$

32. $(6 - 2i)(2 - 3i)$

40. $(1 - 2i)^2 - (1 + 2i)^2$

Complex Conjugates

In Exercises 41–48, write the complex conjugate of the complex number. Then multiply the number by its complex conjugate.

48. $1 + \sqrt{8}$

In Exercises 49–58, write the quotient in standard form.

58. $\dfrac{5i}{(2 + 3i)^2}$

54. $\dfrac{6 - 7i}{1 - 2i}$

In Exercises 59–62, perform the operation and write the result in standard form.

62. $\dfrac{1+i}{i} - \dfrac{3}{4-i}$

Principal Square Root of a Negative Number

If a is a positive number, the **principal square root** of the negative number $-a$ is defined as

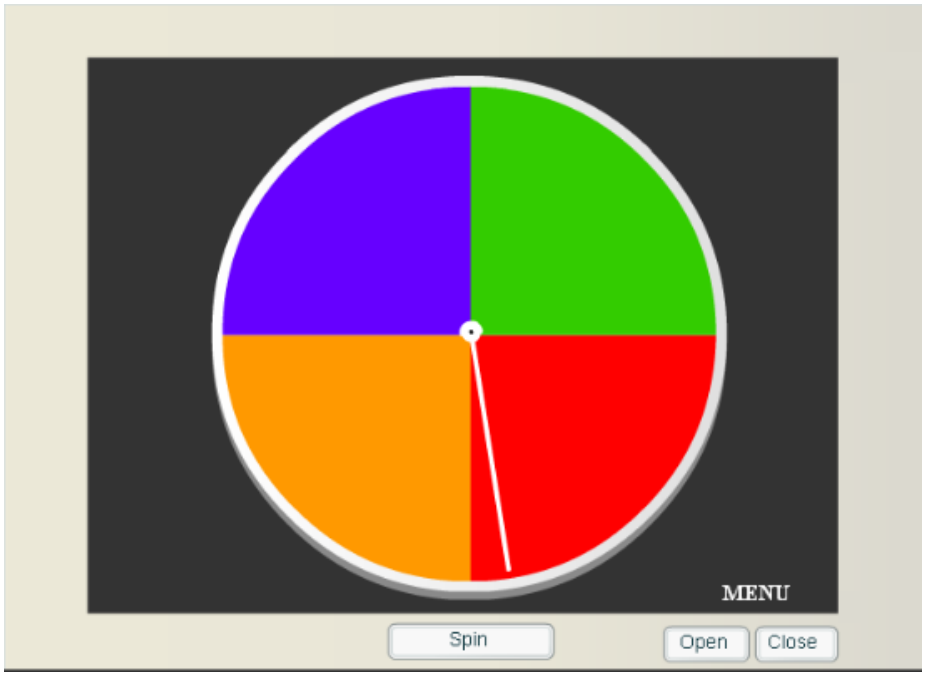
$$\sqrt{-a} = \sqrt{a}i.$$

In Exercises 63–72, use the Quadratic Formula to solve the quadratic equation.

70. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$

In Exercises 73–80, simplify the complex number and write it in standard form.

74. $4i^2 - 2i^3$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