1.8 Other Types of Inequalities

Polynomial Inequalities

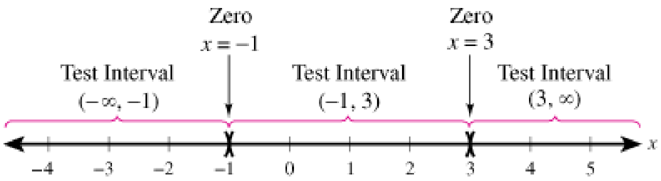
To solve a polynomial inequality such as $x^2 - 2x - 3 < 0$, you can use the fact that a polynomial can change signs only at its zeros (the x-values that make the polynomial equal to zero). Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **critical numbers** of the inequality, and the resulting intervals are the **test intervals** for the inequality. For instance, the polynomial above factors as

$$x^2 - 2x - 3 = (x + 1)(x - 3)$$

and has two zeros, $x = -1$ and $x = 3$. These zeros divide the real number line into three test intervals:

$$(-\infty, -1), \quad (-1, 3), \quad \text{and} \quad (3, \infty). \qquad \text{(See Figure 29.)}$$

So, to solve the inequality $x^2 - 2x - 3 < 0$, you need only test one value from each of these test intervals to determine whether the value satisfies the original inequality. If so, you can conclude that the interval is a solution of the inequality.



Finding Test Intervals for a Polynomial

To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order (from smallest to largest). These zeros are the critical numbers of the polynomial.
2. Use the critical numbers of the polynomial to determine its test intervals.
3. Choose one representative x-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every x-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every x-value in the interval.

In Exercises 1–4, determine whether each value of x is a solution of the inequality.

2. $x^2 - x - 12 \geq 0$

(a) $x = 5$ (b) $x = 0$

(c) $x = -4$ (d) $x = -3$

In Exercises 5–8, find the critical numbers.

6. $9x^3 - 25x^2$

8. $\frac{x}{x+2} - \frac{2}{x-1}$

In Exercises 9–24, solve the inequality and graph the solution on the real number line.

10. $x^2 < 36$

12. $(x - 3)^2 \geq 1$

Rational Inequalities

In Exercises 35–48, solve the inequality and graph the solution on the real number line.

42. $\dfrac{5}{x-6} \geq \dfrac{3}{x+2}$

Graphical Analysis In Exercises 49–52, use a graphing utility to graph the equation. Use the graph to approximate the values of x that satisfy each inequality.

52. $y = \dfrac{5x}{x^2 + 4}$ (a) $y \geq 1$ (b) $y \leq 0$