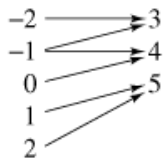


## 2.2 Functions

In Exercises 1–4, is the relationship a function?

2. Domain    Range



#### Definition of a Function

A **function**  $f$  from a set  $A$  to a set  $B$  is a relation that assigns to each element  $x$  in the set  $A$  exactly one element  $y$  in the set  $B$ . The set  $A$  is the **domain** (or set of inputs) of the function  $f$ , and the set  $B$  contains the **range** (or set of outputs).



### Characteristics of a Function from Set $A$ to Set $B$

1. Each element in  $A$  must be matched with an element in  $B$ .
2. Some elements in  $B$  may not be matched with any element in  $A$ .
3. Two or more elements in  $A$  may be matched with the same element in  $B$ .
4. An element in  $A$  (the domain) cannot be matched with two different elements of  $B$ .

### **Four Ways to Represent a Function**

1. *Verbally* by a sentence that describes how the input variable is related to the output variable
2. *Numerically* by a table or a list of ordered pairs that matches input values with output values
3. *Graphically* by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis
4. *Algebraically* by an equation in two variables

## Function Notation

In Exercises 9 and 10, which sets of ordered pairs represent functions from  $A$  to  $B$ ? Explain.

10.  $A = \{a, b, c\}$  and  $B = \{0, 1, 2, 3\}$

- (a)  $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$
- (b)  $\{(a, 1), (b, 2), (c, 3)\}$
- (c)  $\{(1, a), (0, a), (2, c), (3, b)\}$
- (d)  $\{(c, 0), (b, 0), (a, 3)\}$

In Exercises 13–22, determine whether the equation represents  $y$  as a function of  $x$ .

14.  $x = y^2$

20.  $y = \sqrt{x + 5}$

22.  $|y| = 4 - x$

In Exercises 23–36, evaluate the function at each specified value of the independent variable and simplify.

$$34. f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

- (a)  $f(-2)$       (b)  $f(1)$       (c)  $f(2)$

In Exercises 37–42, complete the table.

42. 
$$h(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$$

$x$	$h(x)$
1	
2	
3	
4	
5	

In Exercises 43–50, find all real values of  $x$  such that  $f(x) = 0$ .

46.  $f(x) = \frac{12 - x^2}{5}$

In Exercises 51–54, find the value(s) of  $x$  for which  $f(x) = g(x)$ .

54.  $f(x) = \sqrt{x} - 4$ ,  $g(x) = 2 - x$

## The Domain of a Function

In Exercises 55–68, find the domain of the function.

68.  $f(x) = \frac{x - 5}{\sqrt{x^2 - 9}}$

In Exercises 69–72, assume that the domain of  $f$  is the set  $A = \{-2, -1, 0, 1, 2\}$ . Determine the set of ordered pairs that represents the function  $f$ .

72.  $f(x) = |x + 1|$

In Exercises 77–84, find the difference quotient and simplify your answer.

80.  $f(x) = 4x^2 - 2x$ ,  $\frac{f(x+h) - f(x)}{h}, h \neq 0$

## Applications

90. **Geometry** A rectangle is bounded by the  $x$ -axis and the semicircle  $y = \sqrt{36 - x^2}$  (see figure). Write the area  $A$  of the rectangle as a function of  $x$ , and determine the domain of the function.

### Summary of Function Terminology

*Function:* A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

*Function Notation:*  $y = f(x)$

$f$  is the *name* of the function.

$y$  is the **dependent variable**.

$x$  is the **independent variable**.

$f(x)$  is the *value of the function at  $x$* .

*Domain:* The **domain** of a function is the set of all values (inputs) of the independent variable for which the function is defined. If  $x$  is in the domain of  $f$ ,  $f$  is said to be *defined* at  $x$ . If  $x$  is not in the domain of  $f$ ,  $f$  is said to be *undefined* at  $x$ .

*Range:* The range of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

*Implied Domain:* If  $f$  is defined by an algebraic expression and the domain is not specified, the **implied domain** consists of all real numbers for which the expression is defined.