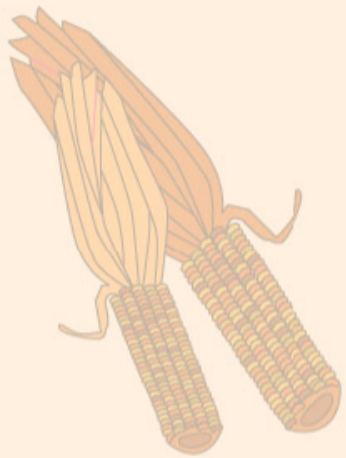
2.3 Analyzing Graphs of Functions



The Graph of a Function

Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no *vertical* line intersects the graph at more than one point.

Zeros of a Function

The **zeros of a function** f of x are the x-values for which $f(x) = 0$.

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval, $f(x_1) = f(x_2)$.

Definition of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

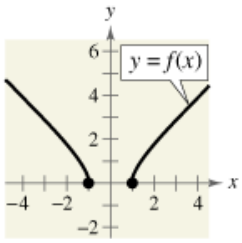
$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \leq f(x).$$

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \geq f(x).$$

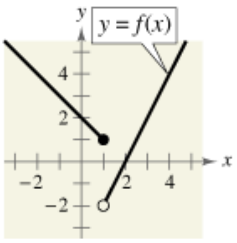
In Exercises 1–4, use the graph of the function to find the domain and range of *f*.

1.



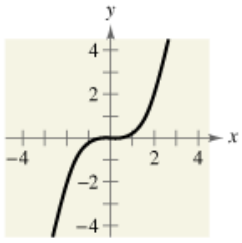
In Exercises 5–8, use the graph of the function to find the indicated function values.

8. (a) $f(2)$ (b) $f(1)$
(c) $f(3)$ (d) $f(-1)$

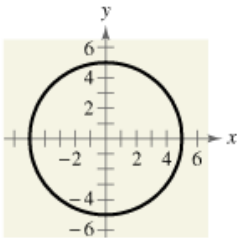


In Exercises 9–14, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, select the MathGraph button.

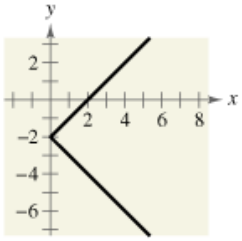
10. $y = \frac{1}{4}x^3$



12. $x^2 + y^2 = 25$



14. $x = |y + 2|$



In Exercises 15–24, find the zeros of the function algebraically.

18. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$

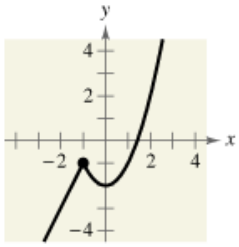
24. $f(x) = \sqrt{3x + 2}$

In Exercises 25–30, use a graphing utility to graph the function and find the zeros of the function. Verify your results algebraically.

28. $f(x) = \sqrt{3x - 14} - 8$

In Exercises 31–38, determine the intervals over which the function is increasing, decreasing, or constant.

36. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$



In Exercises 39–48, (a) use a graphing utility to graph the function and visually determine the intervals over which the function is increasing, decreasing, or constant, and (b) make a table of values to verify whether the function is increasing, decreasing, or constant over the intervals you identified in part (a).

44. $f(x) = 3x^4 - 6x^2$

In Exercises 49–52, use a graphing utility to approximate the relative minimum/relative maximum of each function.

50. $f(x) = 3x^2 - 2x - 5$

In Exercises 53–60, graph the function and determine the interval(s) for which $f(x) \geq 0$.

60. $f(x) = \frac{1}{2}(2 + |x|)$

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** if, for each x in the domain of f,

$$f(-x) = f(x).$$

A function $y = f(x)$ is **odd** if, for each x in the domain of f,

$$f(-x) = -f(x).$$

In Exercises 61–66, determine whether the function is even, odd, or neither.

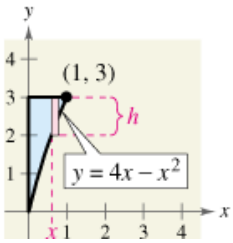
62. $h(x) = x^3 - 5$

64. $f(x) = x\sqrt{1 - x^2}$

66. $g(s) = 4s^{2/3}$

In Exercises 67–70, write the height h of the rectangle as a function of x.

68.



In Exercises 71–74, write the length L of the rectangle as a function of y.

72.