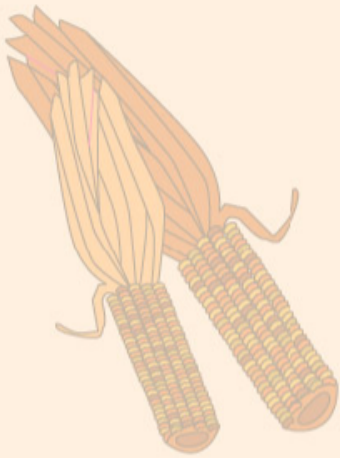
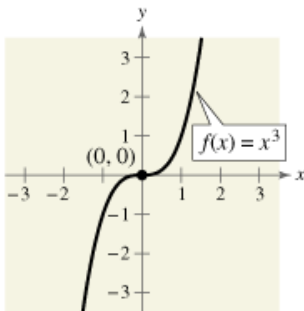


2.4 A Library of Functions



Linear and Squaring Functions

Cubic, Square Root, and Reciprocal Functions



Cubic function
FIGURE 40

y

... are summarized below.

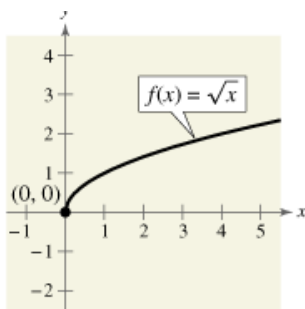
1. The graph of the *cubic* function

$$f(x) = x^3$$

has the following features.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all real numbers.
- The function is odd.
- The graph has an intercept at $(0, 0)$.
- The graph is increasing on the interval $(-\infty, \infty)$.
- The graph is symmetric with respect to the origin.

The graph of the cubic function is shown in Figure 40.



Square root function

2. The graph of the *square root* function

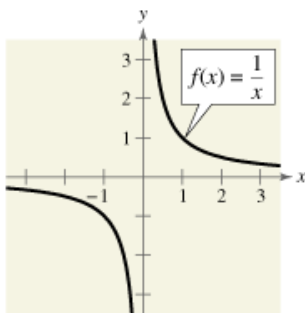
$$f(x) = \sqrt{x}$$

has the following features.

- The domain of the function is the set of all nonnegative real numbers.
- The range of the function is the set of all nonnegative real numbers.
- The graph has an intercept at $(0, 0)$.
- The graph is increasing on the interval $(0, \infty)$.

The graph of the square root function is shown in Figure 41.

FIGURE 41



Reciprocal function

FIGURE 42

3. The graph of the *reciprocal* function

$$f(x) = \frac{1}{x}$$

has the following features.

- The domain of the function is $(-\infty, 0) \cup (0, \infty)$.
- The range of the function is $(-\infty, 0) \cup (0, \infty)$.
- The function is odd.
- The graph does not have any intercepts.
- The graph is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$.
- The graph is symmetric with respect to the origin.

The graph of the reciprocal function is shown in Figure 42.

Step and Piecewise-Defined Functions

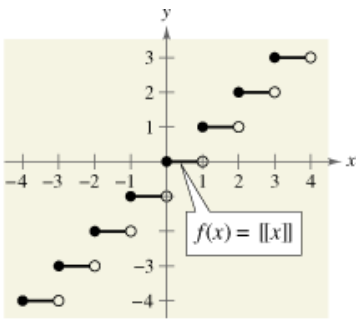


FIGURE 43

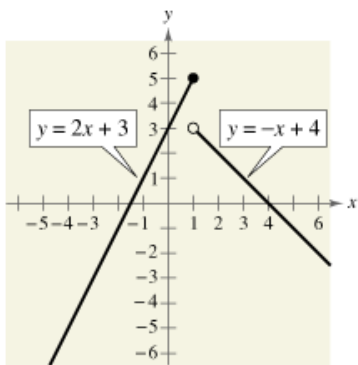


FIGURE 45

Example 3 ▶ Graphing a Piecewise-Defined Function

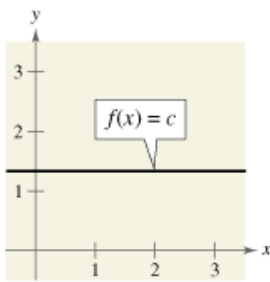
Sketch the graph of

$$f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$$

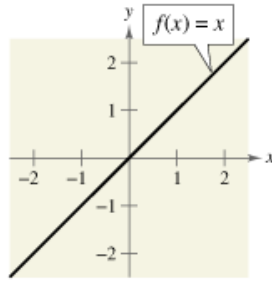
Solution

This piecewise-defined function is composed of two linear functions. At $x = 1$ and to the left of $x = 1$ the graph is the line $y = 2x + 3$, and to the right of $x = 1$ the graph is the line $y = -x + 4$, as shown in Figure 45.

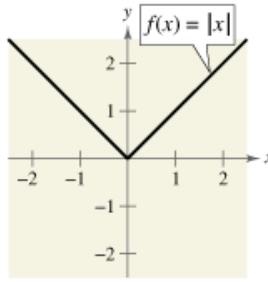
Common Functions



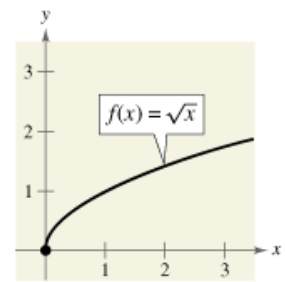
(a) Constant Function



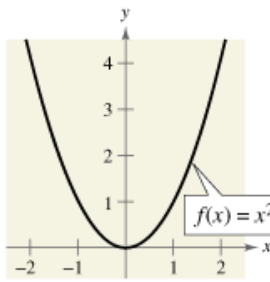
(b) Identity Function



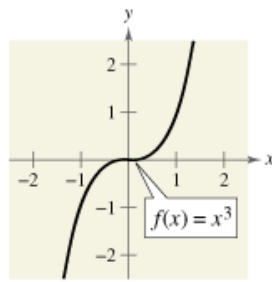
(c) Absolute Value Function



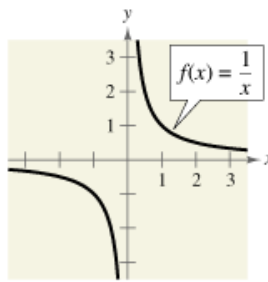
(d) Square Root Function



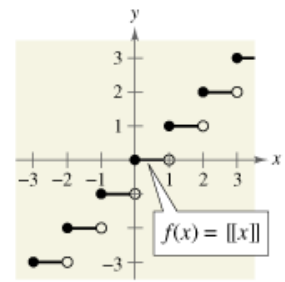
(e) Quadratic Function



(f) Cubic Function



(g) Reciprocal Function



(h) Greatest Integer Function

FIGURE 46

In Exercises 1–8, write the linear function that has the indicated function values.
Then sketch the graph of the function.

6. $f(-10) = 12, f(16) = -1$

In Exercises 9–28, use a graphing utility to graph the function. Be sure to choose an appropriate viewing window.

20. $g(x) = 2(x + 3)^3 + 1$

In Exercise 29–36, evaluate the function for the indicated values.

30. $g(x) = 2\llbracket x \rrbracket$

(a) $g(-3)$ (b) $g(0.25)$ (c) $g(9.5)$ (d) $g\left(\frac{11}{3}\right)$

36. $g(x) = -7\llbracket x + 4 \rrbracket + 6$

(a) $g\left(\frac{1}{8}\right)$ (b) $g(9)$ (c) $g(-4)$ (d) $g\left(\frac{3}{2}\right)$

In Exercises 37–42, sketch the graph of the function.

42. $g(x) = \llbracket x - 3 \rrbracket$

In Exercises 43–50, graph the function.

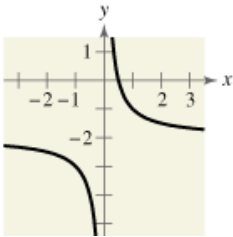
$$50. k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$$

In Exercises 51 and 52, use a graphing utility to graph the function. State the domain and range of the function. Describe the pattern of the graph.

52. $g(x) = 2\left(\frac{1}{4}x - \left\lfloor \frac{1}{4}x \right\rfloor\right)^2$

In Exercises 53–62, identify the common function and the transformed common function shown in the graph. Write an equation for the function shown in the graph. Then use a graphing utility to verify your answer.

56.



omit these

- 67. Wages** A mechanic is paid \$12.00 per hour for regular time and time-and-a-half for overtime. The weekly wage function is

$$W(h) = \begin{cases} 12h, & 0 < h \leq 40 \\ 18(h - 40) + 480, & h > 40 \end{cases}$$

where h is the number of hours worked in a week.

- (a) Evaluate $W(30)$, $W(40)$, $W(45)$, and $W(50)$.
- (b) The company increased the regular work week to 45 hours. What is the new weekly wage function?

