2.5 Shifting, Reflecting, and Stretching Graphs

Exploration

Graphing utilities are ideal tools for exploring translations of functions. Graph f, g, and h in same viewing window. Before looking at the graphs, try to predict how the graphs of g and h relate to the graph of f.

a. $f(x) = x^2$, $g(x) = (x - 4)^2$, $h(x) = (x - 4)^2 + 3$

b. $f(x) = x^2$, $g(x) = (x + 1)^2$, $h(x) = (x + 1)^2 - 2$

c. $f(x) = x^2$, $g(x) = (x + 4)^2$, $h(x) = (x + 4)^2 + 2$

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $h(x) = f(x) + c$

2. Vertical shift c units *downward:* $h(x) = f(x) - c$

3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$

4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$

2. Reflection in the y-axis: $h(x) = f(-x)$

Nonrigid Transformations

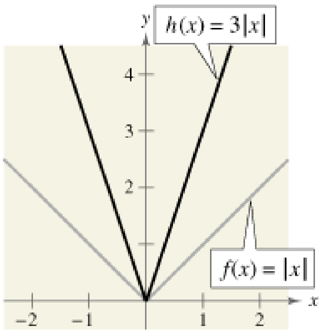


FIGURE 57

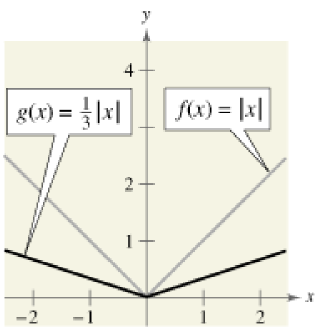


FIGURE 58

Compare the graph of each function with the graph of $f(x) = |x|$.

a. $h(x) = 3|x|$ **b.** $g(x) = \frac{1}{3}|x|$

Solution

a. Relative to the graph of $f(x) = |x|$, the graph of

$$h(x) = 3|x| = 3f(x)$$

is a vertical stretch (each y-value is multiplied by 3) of the graph of f. (See Figure 57.)

b. Similarly, the graph of

$$g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x)$$

is a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$ of the graph of f. (See Figure 58.)

Horizontal shifts, vertical shifts, and reflections are **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the xy-plane. **Nonrigid transformations** are those that cause a *distortion*—a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $g(x) = cf(x)$, where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$. Another nonrigid transformation of the graph of $y = f(x)$ is represented by $h(x) = f(cx)$, where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

$$g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x)$$

is a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$ of the graph of f. (See Figure 58.)

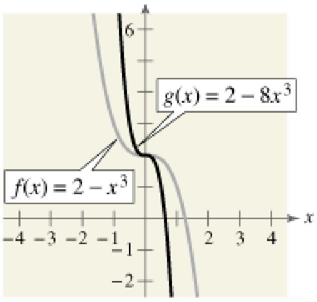


FIGURE 59

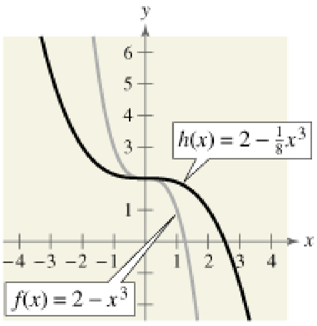


FIGURE 60

Example 5 ▶ Nonrigid Transformations

Compare the graph of each function with the graph of $f(x) = 2 - x^3$.

a. $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$

Solution

a. Relative to the graph of $f(x) = 2 - x^3$, the graph of

$$g(x) = f(2x) = 2 - (2x)^3 = 2 - 8x^3$$

is a horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$ of the graph of f. (See Figure 59.)

b. Similarly, the graph of

$$h(x) = f\left(\tfrac{1}{2}x\right) = 2 - \left(\tfrac{1}{2}x\right)^3 = 2 - \tfrac{1}{8}x^3$$

is a horizontal stretch (each x-value is multiplied by 2) of the graph of f. (See Figure 60.)

In Exercises 39–46, write an equation for the function that is described by the given characteristics.

44. The shape of $f(x) = |x|$, but moved one unit to the left and seven units downward

46. The shape of $f(x) = \sqrt{x}$, but moved nine units downward and reflected in both the x-axis and the y-axis

In Exercises 19–38, describe the transformation from a common function that occurs in the function. Then sketch its graph.

20. $f(x) = (x - 8)^2$

22. $f(x) = -x^3 - 1$

2. For each function sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3.

(a) $f(x) = \sqrt{x} + c$ (b) $f(x) = \sqrt{x - c}$

(c) $f(x) = \sqrt{x - 3} + c$

6. Use the graph of f to sketch each graph. To print an enlarged copy of the graph, select the MathGraph button.

(a) $y = f(-x)$

(b) $y = f(x) + 4$

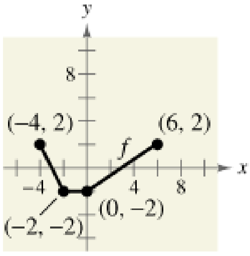
(c) $y = 2f(x)$

(d) $y = -f(x - 4)$

(e) $y = f(x) - 3$

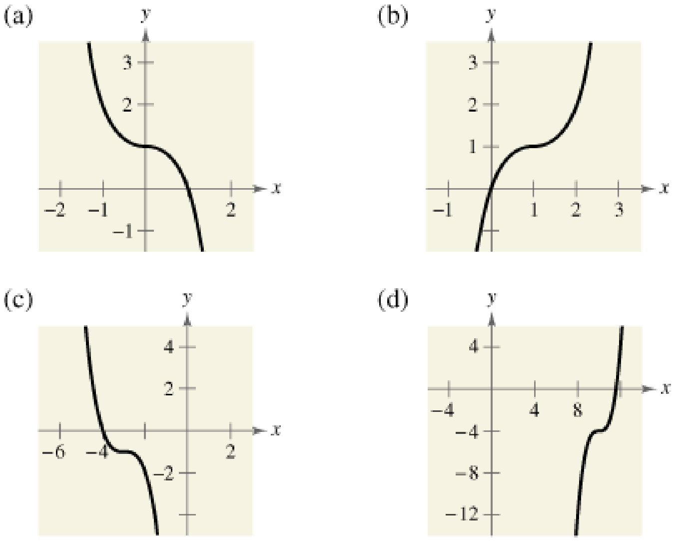
(f) $y = -f(x) - 1$

(g) $y = f(2x)$

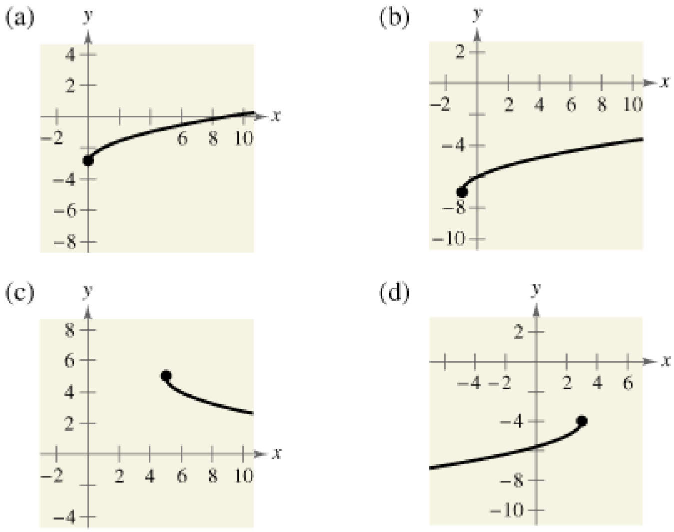


MathGraph

10. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.

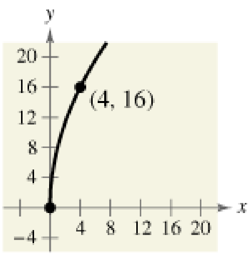


12. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

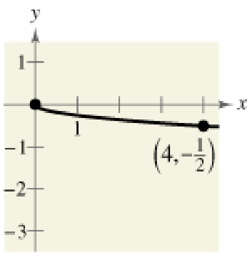


50. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

(a)

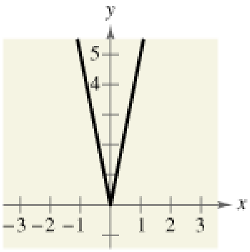


(b)

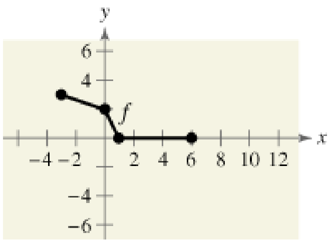


In Exercises 51–56, identify the common function and the transformation shown in the graph. Write an equation for the function shown in the graph. Then use a graphing utility to verify your answer.

52.



62.



MathGraph

(a) $g(x) = f(x) - 5$

(b) $g(x) = f(x) + \frac{1}{2}$

(c) $g(x) = f(-x)$

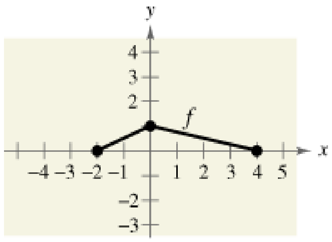
(d) $g(x) = -4f(x)$

(e) $g(x) = f(2x) + 1$

(f) $g(x) = f\left(\frac{1}{4}x\right) - 2$

Graphical Reasoning In Exercises 61 and 62, use the graph of f to sketch the graph of g. To print an enlarged copy of the graph, select the MathGraph button.

61.



MathGraph

(a) $g(x) = f(x) + 2$

(b) $g(x) = f(x) - 1$

(c) $g(x) = f(-x)$

(d) $g(x) = -2f(x)$

(e) $g(x) = f(4x)$

(f) $g(x) = f\left(\frac{1}{2}x\right)$