## 2.5 Shifting, Reflecting, and Stretching Graphs

Snowflakes

**Exploration**

Graphing utilities are ideal tools for exploring translations of functions. Graph $f$, $g$, and $h$ in same viewing window. Before looking at the graphs, try to predict how the graphs of $g$ and $h$ relate to the graph of $f$.

**a.** $f(x) = x^2$, $g(x) = (x - 4)^2$, $h(x) = (x - 4)^2 + 3$

**b.** $f(x) = x^2$, $g(x) = (x + 1)^2$, $h(x) = (x + 1)^2 - 2$

**c.** $f(x) = x^2$, $g(x) = (x + 4)^2$, $h(x) = (x + 4)^2 + 2$

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**Vertical and Horizontal Shifts**

Let $c$ be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift $c$ units *upward:* $h(x) = f(x) + c$
2. Vertical shift $c$ units *downward:* $h(x) = f(x) - c$
3. Horizontal shift $c$ units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift $c$ units to the *left:* $h(x) = f(x + c)$

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### Reflections in the Coordinate Axes

**Reflections** in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the $x$-axis: $h(x) = -f(x)$
2. Reflection in the $y$-axis: $h(x) = f(-x)$

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### Nonrigid Transformations

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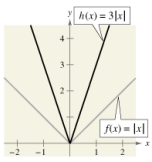


FIGURE 57

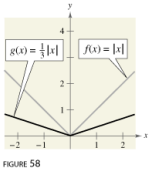


FIGURE 58

Compare the graph of each function with the graph of $f(x) = |x|$.

**a.** $h(x) = 3|x|$ **b.** $g(x) = \frac{1}{3}|x|$

**Solution**

**a.** Relative to the graph of $f(x) = |x|$, the graph of

$$h(x) = 3|x| = 3f(x)$$

is a vertical stretch (each $y$-value is multiplied by 3) of the graph of $f$. (See Figure 57.)

**b.** Similarly, the graph of

$$g(x) = \tfrac{1}{3}|x| = \tfrac{1}{3}f(x)$$

is a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$ of the graph of $f$. (See Figure 58.)

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Horizontal shifts, vertical shifts, and reflections are **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the $xy$-plane. **Nonrigid transformations** are those that cause a *distortion*—a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $g(x) = cf(x)$, where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$. Another nonrigid transformation of the graph of $y = f(x)$ is represented by $h(x) = f(cx)$, where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

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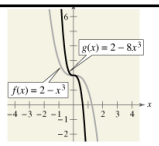


FIGURE 59

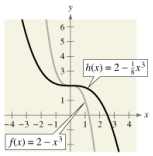


FIGURE 60

$g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x)$

is a vertical shrink (each $y$-value is multiplied by $\frac{1}{3}$) of the graph of $f$. (See Figure 58.)

**Example 5** Nonrigid Transformations

Compare the graph of each function with the graph of $f(x) = 2 - x^3$.

**a.** $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$

**Solution**

**a.** Relative to the graph of $f(x) = 2 - x^3$, the graph of

$$g(x) = f(2x) = 2 - (2x)^3 = 2 - 8x^3$$

is a horizontal shrink (each $x$-value is multiplied by $\frac{1}{2}$) of the graph of $f$. (See Figure 59.)

**b.** Similarly, the graph of

$$h(x) = f\left(\frac{1}{2}x\right) = 2 - \left(\frac{1}{2}x\right)^3 = 2 - \frac{1}{8}x^3$$

is a horizontal stretch (each $x$-value is multiplied by 2) of the graph of $f$. (See Figure 60.)

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**In Exercises 39–46, write an equation for the function that is described by the given characteristics.**

**44.** The shape of $f(x) = |x|$, but moved one unit to the left and seven units downward

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**46.** The shape of $f(x) = \sqrt{x}$, but moved nine units downward and reflected in both the $x$-axis and the $y$-axis

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**In Exercises 19–38, describe the transformation from a common function that occurs in the function. Then sketch its graph.**

**20.** $f(x) = (x - 8)^2$

**22.** $f(x) = -x^3 - 1$

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**2.** For each function sketch (on the same set of coordinate axes) a graph of each function for $c = -3, -1, 1$, and 3.

(a) $f(x) = \sqrt{x} + c$ (b) $f(x) = \sqrt{x - c}$

(c) $f(x) = \sqrt{x - 3} + c$

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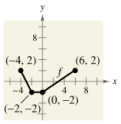
**6.** Use the graph of $f$ to sketch each graph. To print an enlarged copy of the graph, select the MathGraph button.

(a) $y = f(-x)$
(b) $y = f(x) + 4$
(c) $y = 2f(x)$
(d) $y = -f(x - 4)$
(e) $y = f(x) - 3$
(f) $y = -f(x) - 1$
(g) $y = f(2x)$




MathGraph

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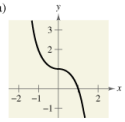
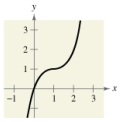
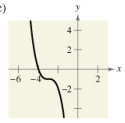
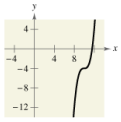
**10.** Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown.

(a)



(b)



(c)



(d)
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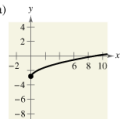
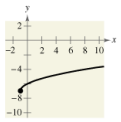
**12.** Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

(a)



(b)
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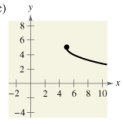
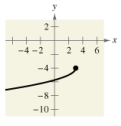



(d)
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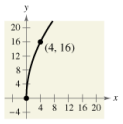
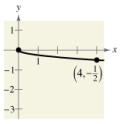
**50.** Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown.

(a)




(b)

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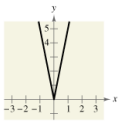
**In Exercises 51–56, identify the common function and the transformation shown in the graph. Write an equation for the function shown in the graph. Then use a graphing utility to verify your answer.**

**52.**

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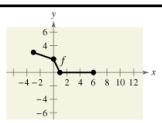
**62.**




MathGraph

(a) $g(x) = f(x) - 5$
(b) $g(x) = f(x) + \frac{1}{2}$
(c) $g(x) = f(-x)$
(d) $g(x) = -4f(x)$
(e) $g(x) = f(2x) + 1$
(f) $g(x) = f\left(\frac{1}{4}x\right) - 2$

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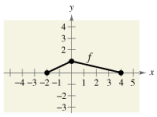
***Graphical Reasoning*** In Exercises 61 and 62, use the graph of $f$ to sketch the graph of $g$. To print an enlarged copy of the graph, select the MathGraph button.

**61.**




MathGraph

(a) $g(x) = f(x) + 2$
(b) $g(x) = f(x) - 1$
(c) $g(x) = f(-x)$
(d) $g(x) = -2f(x)$
(e) $g(x) = f(4x)$
(f) $g(x) = f\left(\frac{1}{2}x\right)$

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