2.6 Combinations of Functions

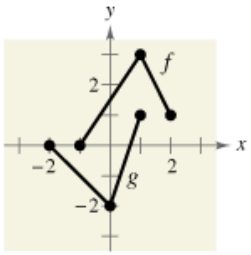
Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum*, *difference*, *product*, and *quotient* of f and g are defined as follows.

1. *Sum:* $(f + g)(x) = f(x) + g(x)$
2. *Difference:* $(f - g)(x) = f(x) - g(x)$
3. *Product:* $(fg)(x) = f(x) \cdot g(x)$
4. *Quotient:* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

In Exercises 1–4, use the graphs of f and g to graph $h(x) = (f + g)(x)$. To print an enlarged copy of the graph, select the MathGraph button.

4.



In Exercises 5–12, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

10. $f(x) = \sqrt{x^2 - 4}, \qquad g(x) = \dfrac{x^2}{x^2 + 1}$

In Exercises 13–24, evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

22. $\left(\frac{f}{g}\right)(0)$

In Exercises 25–28, graph the functions *f*, *g*, and $f + g$ on the same set of coordinate axes.

28. $f(x) = 4 - x^2, \qquad g(x) = x$

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

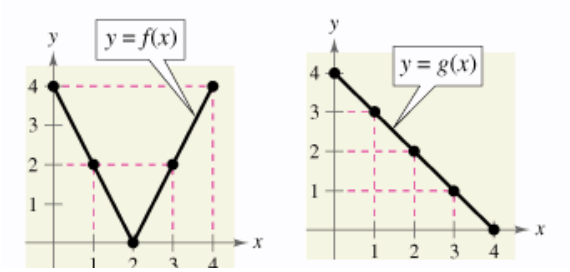
The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 61.)

In Exercises 39–46, find (a) $f \circ g$ and (b) $g \circ f$. Find the domain of each function and each composite function.

46. $f(x) = \dfrac{3}{x^2 - 1}, \qquad g(x) = x + 1$

In Exercises 47–50, use the graphs of *f* and *g* to evaluate the functions.

48. (a) $(f - g)(1)$ (b) $(fg)(4)$



In Exercises 51–58, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There is more than one correct answer.)

52. $h(x) = (1 - x)^3$

58. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

59. ***Geometry*** A square concrete foundation is prepared as a base for a cylindrical tank (see figure).

(a) Write the radius r of the tank as a function of the length x of the sides of the square.

(b) Write the area A of the circular base of the tank as a function of the radius r.

(c) Find and interpret $(A \circ r)(x)$.