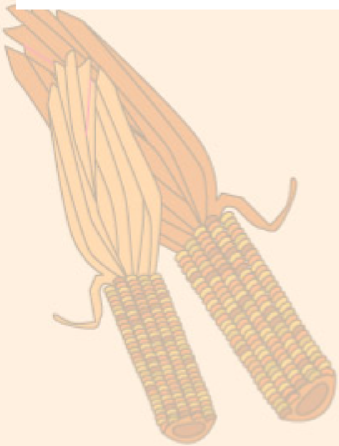
3.4 Concavity and the Second Derivative Test

Definition of Concavity

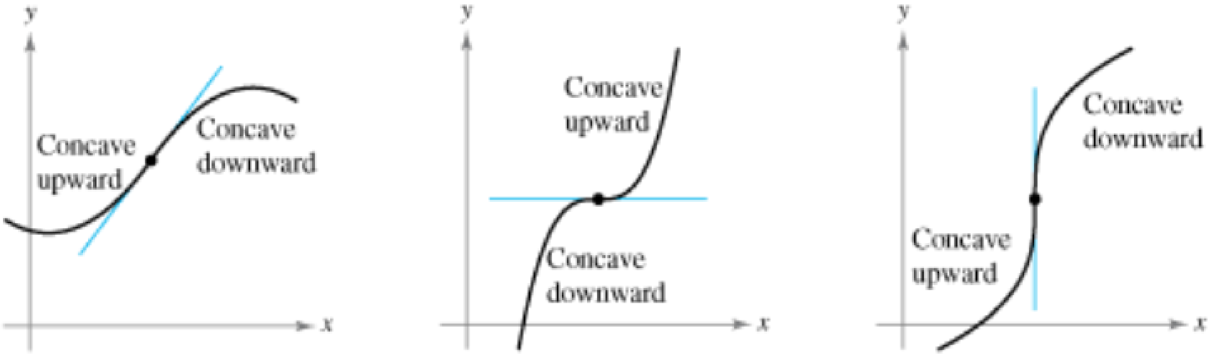
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.



THEOREM 3.7 Test for Concavity

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.



The concavity of f changes at a point of inflection.

THEOREM 3.8 Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c) = 0$ or f is not differentiable at $x = c$.

In Exercises 11–26, find the points of inflection and discuss the concavity of the graph of the function.

16. $f(x) = x^3(x - 4)$

THEOREM 3.9 Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a relative minimum.
2. If $f''(c) < 0$, then $f(c)$ is a relative maximum.

If $f''(c) = 0$, the test fails. In such cases, you can use the First Derivative Test.

In Exercises 27–40, find all relative extrema. Use the Second Derivative Test where applicable.

34. $g(x) = -\frac{1}{8}(x + 2)^2(x - 4)^2$

Think About It **In Exercises 53–56, sketch the graph of a function f having the indicated characteristics.**

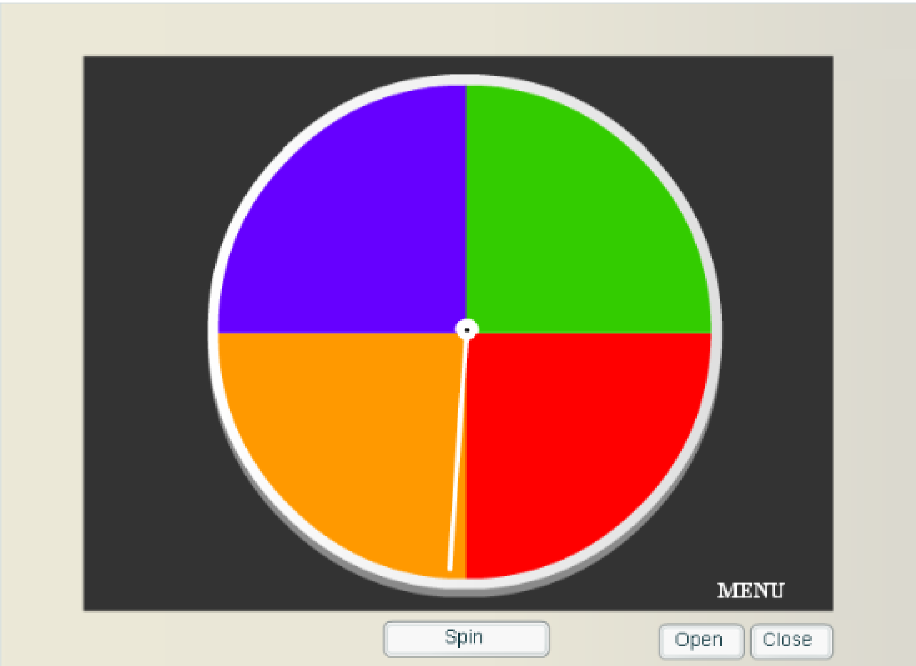
54. $f(0) = f(2) = 0$

$f'(x) > 0$ if $x < 1$

$f'(1) = 0$

$f'(x) < 0$ if $x > 1$

$f''(x) < 0$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