3.5 Limits at Infinity

Definition of a Horizontal Asymptote

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L.$$

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THEOREM 3.10 Limits at Infinity

If r is a positive rational number and c is any real number, then

$$\lim_{x \to \infty} \frac{c}{x^r} = 0.$$

Furthermore, if x^r is defined when $x < 0$, then

$$\lim_{x \to -\infty} \frac{c}{x^r} = 0.$$

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Guidelines for Finding Limits at Infinity of Rational Functions

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

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Definition of Infinite Limits at Infinity

Let f be a function defined on the interval (a, ∞).

1. The statement $\lim_{x \to \infty} f(x) = \infty$ means that for each positive number M, there is a corresponding number $N > 0$ such that $f(x) > M$ whenever $x > N$.
2. The statement $\lim_{x \to \infty} f(x) = -\infty$ means that for each negative number M, there is a corresponding number $N > 0$ such that $f(x) < M$ whenever $x > N$.

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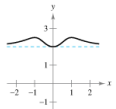
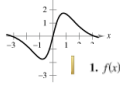
(a)



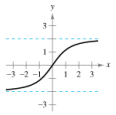
In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.



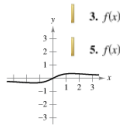
1. $f(x) = \dfrac{3x^2}{x^2 + 2}$

2. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

(c)



(d)



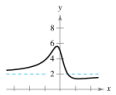
3. $f(x) = \dfrac{x}{x^2 + 2}$

4. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

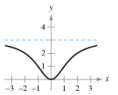
5. $f(x) = \dfrac{4 \sin x}{x^2 + 1}$

6. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

(e)



(f)



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In Exercises 15–18, find each of the limits, if possible.

16. (a) $\lim_{x \to \infty} \dfrac{3 - 2x}{3x^3 - 1}$

(b) $\lim_{x \to \infty} \dfrac{3 - 2x}{3x - 1}$

(c) $\lim_{x \to \infty} \dfrac{3 - 2x^2}{3x - 1}$

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In Exercises 19–32, find the limit.

20. $\lim_{x \to \infty} \dfrac{3x^3 + 2}{9x^3 - 2x^2 + 7}$

22. $\lim_{x \to \infty} \left(4 + \dfrac{3}{x}\right)$

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26. $\lim_{x \to -\infty} \dfrac{x}{\sqrt{x^2 + 1}}$

28. $\lim_{x \to -\infty} \dfrac{-3x + 1}{\sqrt{x^2 + x}}$

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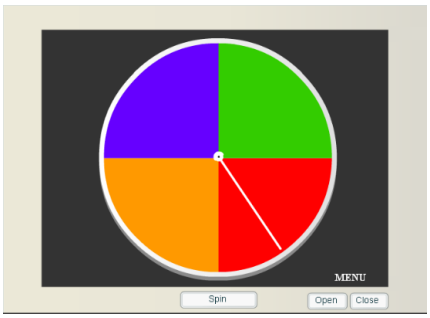

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