

## 3.6 A Summary of Curve Sketching

- $x$ -intercepts and  $y$ -intercepts (Section P.1)
- Symmetry (Section P.1)
- Domain and range (Section P.3)
- Continuity (Section 1.4)
- Vertical asymptotes (Section 1.5)
- Differentiability (Section 2.1)
- Relative extrema (Section 3.1)
- Concavity (Section 3.4)
- Points of inflection (Section 3.4)
- Horizontal asymptotes (Section 3.5)
- Infinite limits at infinity (Section 3.5)



### **Guidelines for Analyzing the Graph of a Function**

1. Determine the domain and range of the function.
2. Determine the intercepts, asymptotes, and symmetry of the graph.
3. Locate the  $x$ -values for which  $f'(x)$  and  $f''(x)$  are either zero or do not exist. Use the results to determine relative extrema and points of inflection.

## Example 1

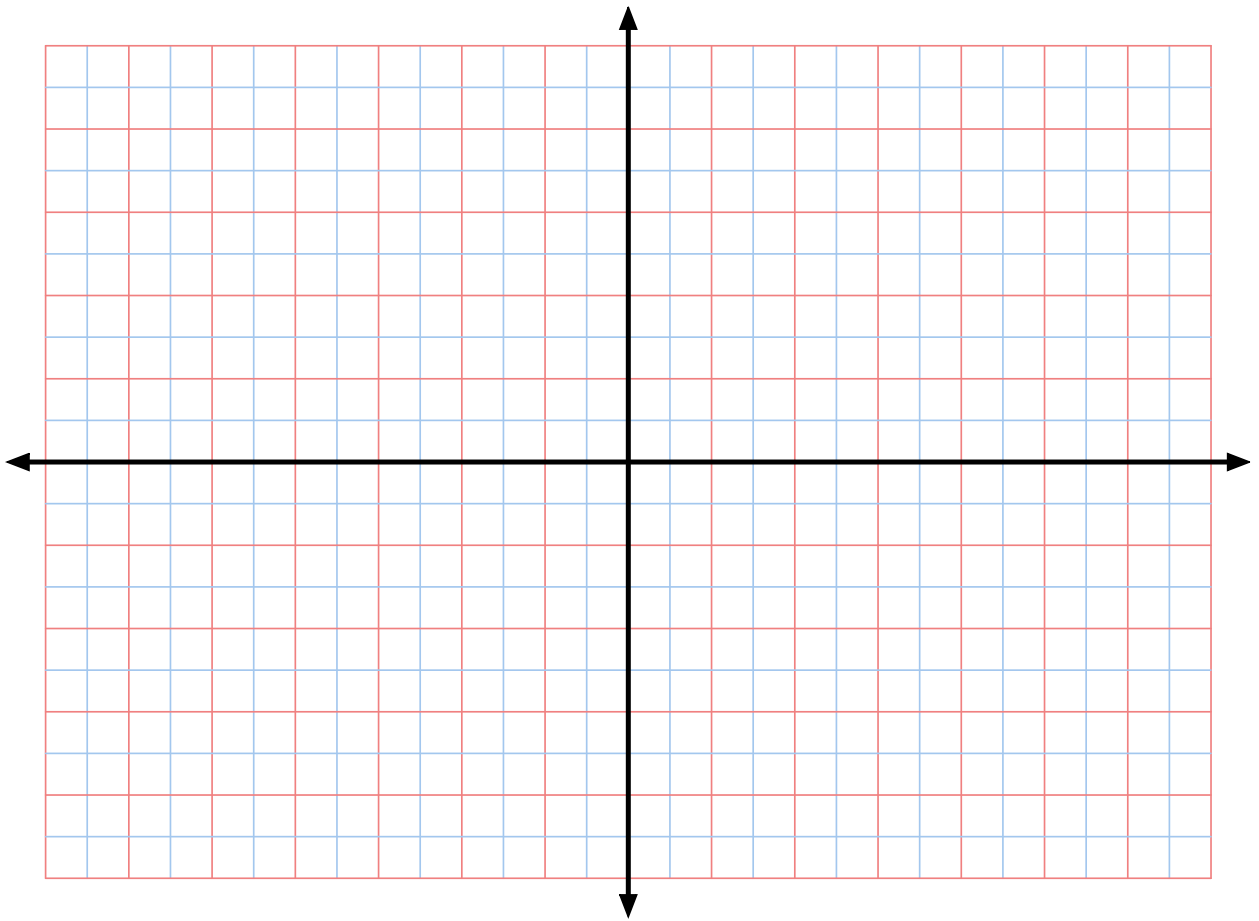
Analyze and sketch the graph of  $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$ .

**This means analyze and graph perfectly without a calculator - on graph paper.**

factors) has a **slant asymptote** if the degree of the numerator exceeds the degree of the denominator by 1. To find the slant asymptote, use long division to rewrite the rational function as the sum of a first-degree polynomial and another rational function.

$$f(x) = \frac{x^2 - 2x + 4}{x - 2} \quad \text{Rewrite using long division.}$$

$$= x + \frac{4}{x - 2} \quad y = x \text{ is a slant asymptote.}$$



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**Example 2** Analyzing and Sketching the Graph of a Rational Function

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Analyze and sketch the graph of  $f(x) = \frac{x^2 - 2x + 4}{x - 2}$ .

