3.9 Differentials

Linear Approximations

graph of a function can be approximated by a straight line.

In Exercises 1–6, find the equation of the tangent line T to the graph of f at the indicated point. Use this linear approximation to complete the table.

x	1.9	1.99	2	2.01	2.1
$f(x)$					
$T(x)$					

2. $f(x) = \dfrac{6}{x^2}$ $\left(2, \dfrac{3}{2}\right)$

Differentials

in x, and is denoted by Δx, as shown in Figure 3.66. When Δx is small, the change in y (denoted by Δy) can be approximated as follows.

$$\Delta y = f(c + \Delta x) - f(c) \qquad \text{Actual change in } y$$
$$\approx f'(c)\Delta x \qquad \text{Approximate change in } y$$

For such an approximation, the quantity Δx is traditionally denoted by dx, and is called the **differential of x.** The expression $f'(x)\,dx$ is denoted by dy, and is called the **differential of y.**

In Exercises 7–10, use the information to evaluate and compare Δy and dy.

7. $y = \frac{1}{2}x^3$ $\quad x = 2 \quad \Delta x = dx = 0.1$

8. $y = 1 - 2x^2$ $\quad x = 0 \quad$ $\Delta x = dx = -0.1$

Differentials

Definition of Differentials

Let $y = f(x)$ represent a function that is differentiable in an open interval containing x. The **differential of x** (denoted by dx) is any nonzero real number. The **differential of y** (denoted by dy) is

$$dy = f'(x)\,dx.$$

In Exercises 11–20, find the differential dy of the given function.

12. $y = 3x^{2/3}$

Error Propagation

Physicists and engineers tend to make liberal use of the approximation of Δy by dy. One way this occurs in practice is in the estimation of errors propagated by physical measuring devices. For example, if you let x represent the measured value of a variable and let $x + \Delta x$ represent the exact value, then Δx is the *error in measurement*. Finally, if the measured value x is used to compute another value $f(x)$, the difference between $f(x + \Delta x)$ and $f(x)$ is the **propagated error.**

$$\underbrace{f(x + \overbrace{\Delta x}^{\text{Measurement error}})}_{\text{Exact value}} - \underbrace{f(x)}_{\text{Measured value}} = \overbrace{\Delta y}^{\text{Propagated error}}$$

32. ***Volume and Surface Area*** The measurement of the edge of a cube is found to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible propagated error in computing

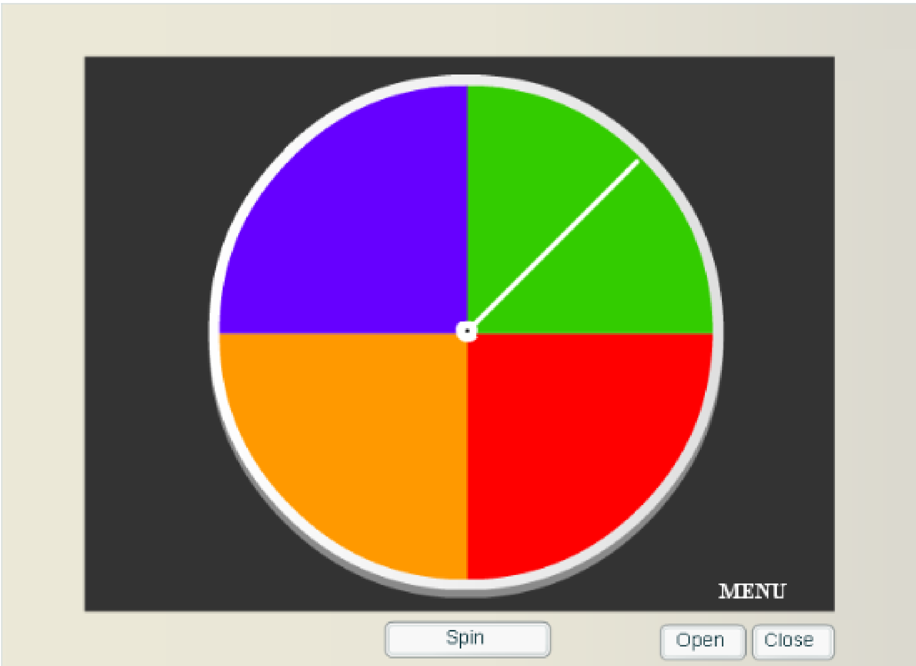
(a) the volume of the cube.

(b) the surface area of the cube.

In Exercises 45–48, use differentials to approximate the value of the expression. Compare your answer with that of a calculator.

45. $\sqrt{99.4}$

46. $\sqrt[3]{26}$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