

The width of the largest subinterval of a partition Δ is the **norm** of the partition and is denoted by $|\Delta|$. If every subinterval is of equal width, the partition is **regular** and the norm is denoted by

$$|\Delta| = \Delta x = \frac{b-a}{n}, \quad \text{Regular partition}$$

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Definition of a Definite Integral

If f is defined on the closed interval $[a, b]$ and the limit

$$\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i$$

exists (as described above), then f is **integrable** on $[a, b]$ and the limit is denoted by

$$\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx.$$

The limit is called the **definite integral** of f from a to b . The number a is the **lower limit** of integration, and the number b is the **upper limit** of integration.

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It is not a coincidence that the notation for definite integrals is similar to that used for indefinite integrals. You will see why in the next section when we discuss the Fundamental Theorem of Calculus. For now it is important to see that definite integrals and indefinite integrals are different identities. A definite integral is a *number*, whereas an indefinite integral is a *family of functions*.

A sufficient condition for a function f to be integrable on $[a, b]$ is that it is continuous on $[a, b]$. A proof of this theorem is beyond the scope of this text.

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Definition of Two Special Definite Integrals

1. If f is defined at $x = a$, then $\int_a^a f(x) dx = 0$.
2. If f is integrable on $[a, b]$, then $\int_b^a f(x) dx = -\int_a^b f(x) dx$.

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THEOREM 4.6 Additive Interval Property

If f is integrable on the three closed intervals determined by a , b , and c , then

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx.$$

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THEOREM 4.7 Properties of Definite Integrals

If f and g are integrable on $[a, b]$ and k is a constant, then the functions of kf and $f \pm g$ are integrable on $[a, b]$, and

1. $\int_a^b kf(x) dx = k \int_a^b f(x) dx$
2. $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx.$

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