#11 ~ Sect. 7.4: Rational Exponents

We need to review the properties of exponents:

Let m and n represent rational numbers. Assume that no denominator equals 0.

Properties:	Examples:
$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^6 = x^{4+6} = x^{10}$
$\left(a^m\right)^n = a^{mn}$	$\left(x^3\right)^7 = x^{3\cdot 7} = x^{21}$
$(ab)^m = a^m b^m$	$(xy)^5 = x^5 y^5$
$a^{-m} = \dfrac{1}{a^m}$	$x^{-9} = \dfrac{1}{x^9}$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^{12}}{x^9} = x^{12-9} = x^3$
$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{x}{y}\right)^6 = \dfrac{x^6}{y^6}$

Radical Form:		Exponential Form:
$\sqrt{25}$	=	$5^{\frac{1}{2}}$
$\sqrt[3]{27}$	=	$27^{\frac{1}{3}}$
$\sqrt[4]{16}$	=	$16^{\frac{1}{4}}$

Ex. 1: Simplify each expression.

a) $64^{\frac{1}{3}}$

b) $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}$

c) $5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}}$

If the nth root of a is a real number and m is an integer, then

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

If m is negative, $a \neq 0$.

<u>Ex. 2:</u>

a) Write each exponential expression in radical form.

1) $x^{\frac{2}{7}}$

2) $y^{-\frac{2}{5}}$

b) Write each radical expression in exponential form.

1) $\sqrt[4]{c^3}$

2) $\left(\sqrt[3]{b}\right)^5$

Ex. 3: Simplify each number.

a) $(-27)^{\frac{2}{3}}$

b) $25^{-2.5}$

Ex. 4: Write each expression in simplest form.

a) $\left(243a^{-10}\right)^{\frac{2}{5}}$

b) $\left(8x^{15}\right)^{-\frac{1}{3}}$