#7 ~ Sect. 7.1: Roots and Radical Expressions

Since 52= 25, 5 is a square root of 25.
Since 53= 125, 5 is a cube root of 125.
Since 54= 625, 5 is a fourth root of 625.
Since 55= 3125, 5 is a fifth root of 3125.
This pattern leads to the definition of nth root.

<u>Definition: nth root</u>

For any real number a and b, and any positive integer n, if a n= b, then a is an <u>nth root</u> of b.

Since 24= 16 and (- 2)4= 16, both 2 and -2 are fourth roots of 16.

Since there is no real number x such that x 4= -16, -16 has no real fourth root.

Since -5 is the only real number whose cube is -125, -5 is the only real cube root of -125.

Some roots, such as the square roots of 10, are irrational numbers. Nevertheless, there is a positive square root and a negative square root of 10.

Here's a summary of the number of possible real roots of a real number:

Type of Number	Number of Real nth Roots When n Is Even	Number of Real nth Roots When n Is Odd
positive	2	1
0	1	1
negative	none	1

Ex. 1: Find all the real roots.

a) the square roots of $0.0001, -1, \frac{36}{121}$.

b) the cube roots of $0.027, -125, \frac{1}{64}$.

c) the fourth roots of $625, -0.0016, \frac{81}{625}$.

$$\sqrt[index]{radicand} = \sqrt[n]{a}$$

- A radical sign is used to indicate a root.
- The number under the radical sign is the radicand.
- The index gives the degree of the root.

When a number has two real roots, the positive root is called the principle root and the radical sign indicates the principle root.

The principle fourth root of 16 is written as $\sqrt[4]{16}$.

The principle fourth root of 16 is 2 because $\sqrt[4]{16}$ equals $\sqrt[4]{2^4}$.

The other fourth root of 16 is written as $-\sqrt[4]{16}$ which equals -2.

Ex. 2: Find each real-number root.

$a)\sqrt[3]{-1000}$ $b)\sqrt{-81}$

Notice that when $x = 5, \sqrt{x^2} = \sqrt{5^2} = \sqrt{25} = 5 = x,$

and when $x = -5, \sqrt{x^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq x.$

Property: nth Root of a n, a<0

For any negative real number a,

$\sqrt[n]{a^n} = |a|$ when n is even.

Ex. 3: Simplify each radical expression.

$a)\sqrt{9x^{10}}$ $b)\sqrt[3]{a^3b^3}$ $c)\sqrt[4]{x^{16}y^4}$

Ex. 4: The weight of an orange is related to its diameter by the formula $w = \frac{d^3}{4}$ where d is the diameter in inches and w is the weight in ounces. Find the diameter of each orange with each weight.

$a) 3oz$

$b) 5.5oz$