

## 5.2 Trig Functions: Unit Circle Approach

**Unit Circle** - radius of 1; center at origin; circumference =  $2\pi$

Let  $t \geq 0$  be any real # and  $s$  be the distance from  $(1,0)$  beginning of circle to  $t$ . Wrap that distance around arc in counterclockwise rotation to get  $p(x,y)$ .

If  $t < 0$  still begin at pt.  $(1,0)$  travel  $s = |t|$  units in clockwise direction. See figure 20b on pg. 383.

So, for any real #  $t$  we can locate a unique point  $(x,y)$  on the unit circle. We call  $p$  "the point on the unit circle that corresponds to  $t$ ". We use  $p$  coordinates on the unit circle to define trig functions for  $t$ .

Sine function:  $\sin t = y$

Cosine:  $\cos t = x$

Tangent:  $\tan t = y/x$

Cosecant:  $\csc t = 1/y$

Secant:  $\sec t = 1/x$

Cotangent:  $\cot = x/y$

If an angle is measured in degrees, use the degree symbol when writing trig functions ( $\sin 30^\circ$ ,  $\tan 45^\circ$ ).

If an angle is measured in radians, then no symbol is used when writing trig functions ( $\cos \pi$ ,  $\sec \pi/3$ )

Since  $\pi/2$  radians =  $90^\circ$   $p = (0,1)$   
 $\therefore \sin \pi/2 = \sin 90^\circ = 1$  and  $\cos \pi/2 = \cos 90^\circ = 0$

Ex. 2 Finding exact values of trig functions of quadrantal angles (terminal side on x or y axis).

a)  $\theta = 0 = 0^\circ$   $p = (1,0)$

b)  $\theta = \pi/2 = 90^\circ$   $p = (0,1)$

Look at chart pg. 387 table 2

Finding exact values of trig functions of  $\pi/4 = 45$

notice that  $p$  lies on the line  $y = x$ , since 45 is  $1/2$  of 90 ...it bisects quadrant I. Since  $p = (x,y)$  also lies on the unit circle,  $x^2 + y^2 = 1$

Ex. 3 Find exact values

a)  $\sin 45 \cos 180$

b)  $(\sec \pi/4)^2 + \csc \pi/2$

Trig functions of  $\pi/6 = 30$  and  $\pi/3 = 60$

Ex. 4 Find

a)  $\sin \pi/3$

b)  $\csc 60$

c)  $\tan \pi/3$

d)  $\cos \pi/6$

e)  $\tan 30$

Exact values of multiples of  $\pi/6, \pi/4, \pi/3$  30 , 60 , 90

- we know  $\pi/4 = 45$  p =

multiples of  $\pi/6$

multiples of  $\pi/3$

Use calculator to find trig values (should have sin, cos, tan keys)

$$\sec = 1/x = 1/\cos \quad \csc = 1/y = 1/\sin \quad \cot = x/y = 1/\tan$$

Ex. 5 Find and round to 2 decimal places

a)  $\cos 48$       b)  $\csc 21$       c)  $\tan \pi/12$

Using a circle of radius  $r$  to evaluate trig functions (not a unit circle)

this changes the equations to:

$$\begin{array}{lll} \sin = y/r & \cos = x/r & \tan = y/x \\ \csc = r/y & \sec = r/x & \cot = x/y \end{array}$$

Ex. Find exact trig values of angle  $\theta$  if  $(4, -3)$  is a point on its terminal side.