5.3 Properties of Trig Functions

Domain & Range

1. Domain of the sine function is the set of real #'s
2. Domain of the cosine function is the set of real #'s

If x = 0 then tangent (y/x) and secant (1/x) functions are undefined. On Unit circle there are 2 points that have x = 0 (π/2 & 3π/2). What are those degree measures?

In general, odd multiples of π/2 (90) such as: π/2, 3π/2, 5π/2 also -π/2 and so on.

3. Domain of the tangent & secant functions are the set of all real #'s, except odd multiples of π/2

If y = 0 then the cotangent (x/y) and cosecant (1/y) functions are undefned. On Unit circle there are 2 points that have y = 0 (0 & π). What are those degree measures?

In general, any angle that is a multiple of π (180) such as: 0, π, 2π, 3π, -π and so on.

4. Domain of the cotangent & cosecant functions are the set of all real #'s, except the multiples of π

Range: Sin & Cos (remember sine & cosine waves....we will review them in 5.4)

$-1 \leq \sin \leq 1$ $-1 \leq \cos \leq 1$

Pg. 400 to see the following:

$\csc \leq -1$ or $\csc \geq 1$

$\sec \leq -1$ or $\sec \geq 1$

$-\infty < \tan < \infty$ $-\infty < \cot < \infty$ (see table 4 on pg. 401)

Period of the Trig Function

If we use $\pi/3$ p = $(1/2, \sqrt{3}/2)$ notice if we write $\pi/3 + 2\pi$, the point would be the same, since 2π is just one full revolution of the unit circle.

Periodic - A function is called periodic if there is a positive # p such that, whenever is in the domain of f, so is + p and:

f(+ p) = f ( )

Periodic Properties:

Ex. 1 Find exact values values of:

a) sin 17π/4 b) cos (5π) c) tan 5π/4

The signs of Trig Functions

P is in quad IV, so x is positive and y is negative, so

Ex. 2 Find which quad the angle lies if sin < 0 & cos < 0

Fundamental Identities

Sin	= y	cos	= x	tan	= y/x
Csc	= 1/y	sec	= 1/x	cot	= x/y

Reciprocal Identities (we have already)

csc	= 1/sin	sec	= 1/cos	cot	= cos /sin

Quotient Identities

tan	= sin /cos	cot	= cos /sin

Ex. 3 Finding exact values using identities when sine & cosine are given
given sin $= \sqrt{5}/5$ cos $= 2\sqrt{5}/5$ find remaining 4 trig values.

Using the equation of the Unit circle $x^2 + y^2 = 1$, if p = (x,y) is the point that corresponds to angle , then $y^2 + x^2 = 1$ since y = sin & x = cos

Other formulas: Pg. 405 for proof

$\tan^2 + 1 = \sec^2$
$1 + \cot^2 = \csc^2$

Ex. 4 Find exact values of trig expressions using identities
a) tan 20 - sin 20 /cos 20 b) $\sin^2 \pi/12 + 1/\sec^2 \pi/12$

Ex. 5 Finding exact values given one value & the sign of another
given sin = 1/3 & cos = x < 0

Ex. 6 Given one value of a trig function, find remaining values.
given tan = 1/2 & sin < 0

Even - Odd Properties

Recall that a function f is even if f(-) = f( ) for all in the domain of f. A function f is odd if f(-) = -f( ) for all in the domain of f.

Ex. 7 Find exact values using Even - Odd properties

a) $\sin(-45^\circ)$ b) $\cos(-\pi)$ c) $\cot(-3\pi/2)$ d) $\tan(-37\pi/4)$