5.4 Graphs of Sine & Cosine functions

Since the sine function has period 2π, we need to graph y = sin x only on the interval [0, 2π]. The remainder of the graph will consist of repetitions of this portion of the graph (goes on forever in both directions).

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x	y=sinx	(x,y)
0	0	(0,0)
π/6	½	(π/6,½)
π/2	1	(π/2,1)
5π/6	½	(5π/6,½)
π	0	(π,0)
7π/6	-½	(7π/6,-½)
3π/2	-1	(3π/2,-1)
11π/6	-½	(11π/6,-½)
2π	0	(2π,0)

The graph is one peripd (cycle) of the graph y = sin x. To obtain more of the graph just repeat this period in each direction

Some things we should know about the Sine from previous sections

- domain: all real #'s
- range: $-1 \leq y \leq 1$
- odd function: symmetric with respect to origin
- periodic: with period 2π
- x-int: ...., -2π, 0, 2π,.......
- y-int: 0
- max value of 1 occurs at x = ..., $-3\pi/2$, $\pi/2$, $5\pi/2$, $9\pi/2$, ...
- min value of -1 occurs at x = ..., $-\pi/2$, $3\pi/2$, $7\pi/2$, $11\pi/2$, ...

Ex. 1 Use the graph $y = \sin x$ to graph $y = \sin(x - \pi/4)$

Ex. 2 Use graph $y = \sin x$ to graph $y = -\sin x + 2$

The Cosine function also has period 2π, so we will proceed just like the Sine function.

x	y=cosx	(x,y)
0	1	(0,1)
π/3	½	(π/3,½)
π/2	0	(π/2,0)
2π/3	-½	(2π/3,-½)
π	-1	(π,-1)
4π/3	-½	(4π/3,-½)
3π/2	0	(3π/2,0)
5π/3	½	(5π/3,½)
2π	1	(2π,1)

Properties of Cosine function that we should already know:

- domain: all real #'s
- range: $-1 \leq y \leq 1$
- even function: symmetric with respect to y-axis
- periodic: with period 2π
- x-int: ....., -3π/2, -π/2, π/2, 3π/2, 5π/2, ...
- y-int: 1
- max value of 1 at x = ..., -2π, 0, 2π, 4π, ...
- min value of -1 at x = ..., -π, π, 3π, 5π, ...

Ex. 3 Use graph y = cos x to graph y = 2 cos x

Ex. 4 Use grah y = cos x to graph y = cos (3x)

Comparing $y = \cos x$ with $y = \sin x$

In ex. 3 we graphed $y = 2 \cos x$; notice that the values of $y = 2 \cos x$ lie between -2 & 2 inclusive.

In general, the values of functions $y = A \sin x$ & $y = A \cos x$ where $A \neq 0$ will always satisfy the inequalities:

$-|A| \leq A \sin x \leq |A|$ & $-|A| \leq A \cos x \leq |A|$

The number $|A|$ is called the amplitude

In ex. 4, we graphed $y = \cos(3x)$ and the period of that function is $2\pi/3$

In general, if $w > 0$, the functions $y = \sin(wx)$ & $y = \cos(wx)$ will have period $T = 2\pi/w$. The graphs are either compressed or stretched by a factor of $1/w$. This compression or stretch replaces the interval $[0, 2\pi]$ which contains one period (cycle) by interval $[0, 2\pi/w]$.

The period of the function $y = \sin(wx)$ & $y = \cos(wx)$ where $w>0$ is $2\pi/w$

- if $w<0$ in $y = \sin(wx)$ or $y = \cos(wx)$ we can use the even-odd properties of sine & cosine.

$\sin(-wx) = -\sin(wx)$

$\cos(-wx) = -\cos(wx)$

ex. $\sin(-2x) =$

ex. $\cos(-\pi x) =$

$w > 0$ Amplitude $= |A|$ Period $= T = 2\pi/w$

Ex. 5 Determine amplitude & period of $y = 3\sin(4x)$

Recall graphs for $y = \sin x$ & $y = \cos x$

One period (cycle) is from [0, 2π] notice that each graph consists of 4 parts corresponding to 4 subintervals [0,π/2], [π/2, π], [π, 3π/2], [3π/2, 2π]. Each subinterval is length π/2 (which is the period 2π divided by 4) and the endpoints of the intervals give rise to 5 key points on the graph.

Whne graphing by hand: use amplitude to determine the max & min values of the function. The period is used to divide the x-axis into 4 subintervals. The endpoints of the subintervals give rise to 5 key points on the graph, which are used to sketch one cycle. Finally, extebd the graph in both directions to make it complete.

***using graphing calc: use amplitude to set the y-max & y-min and use the period to set the x-min & x-max.

Ex. 6 Graph $y = 3 \sin (4x)$ from ex.5 amplitude = 3 & period = $\pi/2$

Ex. 7 If we had $y = 4 \cos(\pi x)$

If we had $y = -4 \cos(\pi x)$

Ex. 8 Finding an equation for a sinusoidal graph

a)

b)