5.5 Graphs of Tangent, Cotangent, Cosecant and Secant

Graphs of y = tan x & y = cot x

Because tan function has period π, we only need to determine the graphs over some interval of length π. Because tan function is not defined at odd multiples π/2 (....., -3π/2, -π/2, π/2, 3π/2,.....) we will use the interval [-π/2, π/2].

table on pg. 430

| x | y = tan x | (x,y) |
|---|---|---|
| -π/3 | | (-π/3, ) |
| -π/4 | -1 | (-π/4, -1) |
| -π/6 | | (-π/6, ) |
| 0 | 0 | (0, 0) |
| π/6 | | (π/6, ) |
| π/4 | 1 | (π/4, 1) |
| π/3 | | (π/3, ) |

The closer x gets to $\pi/2$, the closer sin x gets to 1 and cos x gets to "0". So, tan approaches $\infty$ ($\lim \tan x = \infty$). In other words, the vertical line $x = \pi/2$ is a vertical assymptote to the graph $y = \tan x$.

If x is close to $-\pi/2$, but remains greater than $-\pi/2$ (approaches from the right). then sin x gets close to -1 and cos x will be positive and close to "0". Hence, this ratio sin x / cos x approaches $-\infty$ ($\lim \tan x = -\infty$). $x = -\pi/2$ is also a vertacal assymptote.

With this we can complete the graph $y = \tan x$

$y = \tan x$, $-\infty < x < \infty$ x not defined at odd multiples of $\pi/2$

If using a graphing calculator..........use dot mode.

Facts we should know about $y = \tan x$

- domain: all real #'s, except odd multiples of $\pi/2$
- range: all real #'s
- odd function: symmetric with respect to origin
- periodic: with period $\pi$
- x-int: ....., $-2\pi$, $-\pi$, 0 , $\pi$, $2\pi$,......

y-int: 0

vertical assymptotes at x = ......, $-3\pi/2$, $-\pi/2$, $\pi/2$, $3\pi/2$,.....

Ex. 1 Graph $y = 2 \tan x$

Ex. 2 Graph y = - tan (x + π/4)
use y = tan x graph and shift left π/4 units

We obtain the graph y = cot x as we did for the graph y = tan x. The period of y = cot x is π. Because cot function is not defined for multiples of π, we will use the interval [0, π]. Show unit circle for points.

table on pg. 433

| x | y = cot x | (x,y) |
|---|---|---|
| π/6 | | (π/6, ) |
| π/4 | 1 | (π/4, 1) |
| π/3 | | (π/3, ) |
| π/2 | 0 | (π/2, 0) |
| 2π/3 | | (2π/3, ) |
| 3π/4 | -1 | (3π/4, -1) |
| 5π/6 | | (5π/6, ) |

y = cot x , $-\infty < x < \infty$ x not defined at multiples of π

The graphs of y = csc x & y = sec x (use reciprocal identities)

csc x = 1/sin x          sec x = 1/cos x

y = csc x   $-\infty < x < \infty$   x not equal to multiples of $\pi$ (y-values would be "0")

y = sec x   $-\infty < x < \infty$   x not equal to odd multiples of $\pi/2$ (x-values would be "0")

Ex. 3 Graph $y = 2 \csc (x - \pi/2)$