

## 6.1 Inverse Sine, Cosine & Tangent Functions

### Properties of a function $f$ & its inverse function $f^{-1}$

$f^{-1}(f(x)) = x$  for every  $x$  in the domain of  $f$  and

$f(f^{-1}(x)) = x$  for every  $x$  in the domain of  $f^{-1}$

domain of  $f$  = range of  $f^{-1}$  and range of  $f$  = domain of  $f^{-1}$

\*\*\*remember pt.  $(x,y)$  has inverse of pt.  $(y,x)$

graphs of  $f$  &  $f^{-1}$  are symmetric w/ respect to line  $y = x$

If a function  $y = f(x)$  has an inverse function, the equation of the inverse function is  $x = f^{-1}(y)$ . The solution to this equation is  $y = f(x)$

### Inverse Sine Function

The inverse of  $y = f(x) = \sin x$  is obtained by interchanging  $x$  &  $y$  therefore

$$x = \sin y, -\pi/2 \leq x \leq \pi/2$$

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$$y = \sin^{-1}x \text{ means } x = \sin y$$

where  $-1 \leq x \leq 1$  and  $-\pi/2 \leq y \leq \pi/2$

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$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \text{ where } -\pi/2 \leq x \leq \pi/2$$

$$f(f^{-1}(x)) = \sin(\sin^{-1}x) = x \text{ where } -1 \leq x \leq 1$$

for ex. b/c  $\pi/8$  lies in the interval  $[-\pi/2, \pi/2]$  the restricted domain of the sine function, we have  $\sin^{-1}[\sin(\pi/8)] = \pi/8$

also, b/c  $0.8$  lies in the interval  $[-1, 1]$ , the domain of the sine function, we have  $\sin^{-1}[\sin(0.8)] = 0.8$

Check on graphing calc:  $\sin^{-1}(\sin(\pi/8)) = .3926990817$   
 $\sin(\sin^{-1}(0.8)) = 0.8$

## Inverse Sine

Ex. 1 find exact value of  $\sin^{-1}1$

let  $\theta = \sin^{-1}1$  solve angle  $\theta$ ,  $-\pi/2 \leq \theta \leq \pi/2$ , whose sine equals 1

Ex. 2 find exact value of  $\sin^{-1}(-1/2)$

Ex. 3 find approximate value of (round to 2 decimal places)

a)  $\sin^{-1}(1/3)$                       b)  $\sin^{-1}(-1/4)$

### Inverse Cosine Function

We can restrict domain of  $y = \cos x$  to the interval  $[0, \pi]$

$y = \cos x$ ,  $0 \leq x \leq \pi$  is a one - to - one function so it has an inverse

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$$y = \cos^{-1}x \text{ means } x = \cos y$$
$$-1 \leq x \leq 1 \qquad 0 \leq y \leq \pi$$

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## Inverse Cosine

Ex. 4 find exact value of  $\cos^{-1}0$

Ex. 5 find exact value of  $\cos^{-1}\sqrt{2}/2$

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \text{ where } 0 \leq x \leq \pi$$

$$f(f^{-1}(x)) = \cos(\cos^{-1}x) = x \text{ where } -1 \leq x \leq 1$$

Ex. 6 find exact value of

a)  $\cos^{-1}[\cos(\pi/12)]$

b)  $\cos[\cos^{-1}(-0.4)]$

## Inverse Tangent Function

We can restrict domain of  $y = \tan x$  to the interval  $[-\pi/2, \pi/2]$

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$$\begin{array}{l} y = \tan^{-1}x \quad \text{means} \quad x = \tan y \\ -\infty < x < \infty \quad \quad -\pi/2 < y < \pi/2 \end{array}$$

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Ex. 7 find exact value of  $\tan^{-1} 1$

Ex. 8 find exact value of  $\tan^{-1}(-\sqrt{3})$

Help set up problem #45 on HW assignment