

6.4 Sum & Difference Formulas

Cosine:

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad \text{proof on pg. 483}$$

Ex. 1 Using sum formula to find exact values. Find exact value of 75°

Ex. 2 Using difference formula to find exact values. Find exact value of $\pi/12$

Identity: $\cos(\pi/2 - \theta) = \sin \theta$
 $\sin(\pi/2 - \theta) = \cos \theta$

Sum & difference for sines:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

proof on pg. 485

Ex. 3 Find exact value of $\sin 7\pi/12$

Ex. 4 Find exact value of $\sin 80 \cos 20 - \cos 80 \sin 20$

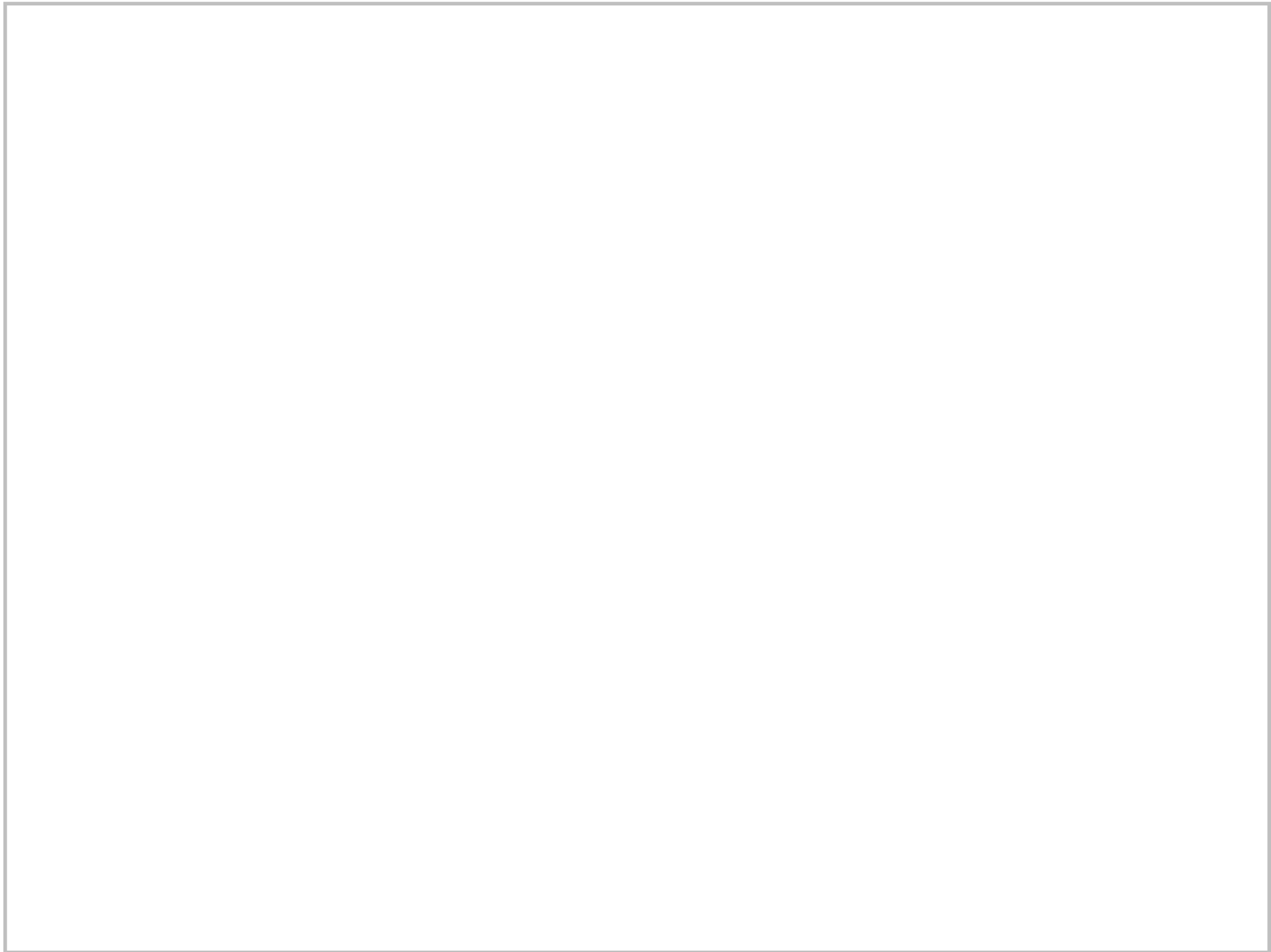
* this is in the form of $\sin(\alpha - \beta)$

Ex. 5 given $\sin \alpha = 4/5$, $\pi/2 < \alpha < \pi$ (which Quad is this?)

$$\sin \beta = -2/\sqrt{5}, \pi < \beta < 3\pi/2$$

Find exact values of:

a) $\cos \alpha$ b) $\cos \beta$ c) $\cos(\alpha + \beta)$ d) $\sin(\alpha + \beta)$



Ex. 6 Establish the identity: $\cos(\alpha - \beta) / \sin \alpha \sin \beta = \cot \alpha \cot \beta + 1$

Sum & Difference formulas for tangents:

$$\tan(\alpha + \beta) = (\tan \alpha + \tan \beta) / (1 - \tan \alpha \tan \beta)$$

$$\tan(\alpha - \beta) = (\tan \alpha - \tan \beta) / (1 + \tan \alpha \tan \beta)$$

* can only be used for angles α, β for which $\tan \alpha$ & $\tan \beta$ are defined, that is, all angles except odd multiples of $\pi/2$.

proof on pg. 488

Ex. 7 Prove identity: $\tan(\theta + \pi) = \tan \theta$, we are proving that the tangent function is periodic w/ period π .

Ex. 8 Prove identity: $\tan(\theta + \pi/2) = -\cot \theta$

Ex. 9 Find exact value of: $\sin(\cos^{-1} 1/2 + \sin^{-1} 3/5)$. This is the sine of the sum of 2 angles. $\alpha = \cos^{-1} 1/2$, $\beta = \sin^{-1} 3/5$