

6.5 Double Angle & Half Angle Formulas

In the sum formulas for $\sin(\alpha + \beta)$ & $\cos(\alpha + \beta)$ let $\alpha = \beta = \theta$.

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\theta + \theta) = \sin \theta \cos \theta + \cos \theta \sin \theta$$

$$**\sin(2\theta) = 2 \sin \theta \cos \theta \quad (1)$$

Using the same process for cos we would get:

$$**\cos(2\theta) = \cos^2 \theta - \sin^2 \theta \quad (2)$$

An application of the pythagorean identity: $\sin^2 \theta + \cos^2 \theta = 1$ results in two other ways to write formula (2): $\cos(2\theta)$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = ??? \quad (3)$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = ??? \quad (4)$$

Ex. 1 if $\sin \theta = 3/5$, $\pi/2 < \theta < \pi$ (quad ?) find exact value of:

a) $\sin (2\theta)$

b) $\cos (2\theta)$

$$**\tan (2\theta) = (2 \tan \theta) / (1 - \tan^2 \theta) \quad (5)$$

Other variations of double angle formulas are:

$$** \sin 2\theta = (1 - \cos 2\theta) / 2 \quad (6)$$

$$** \cos 2\theta = (1 + \cos 2\theta) / 2 \quad (7)$$

$$** \tan 2\theta = (1 - \cos 2\theta) / (1 + \cos 2\theta) \quad (8)$$

Half Angle Formulas

can use formulas 6 thru 8 to prove half angles: let $\theta = \alpha/2$ and we would get:

$$\sin \alpha/2 =$$

$$\cos \alpha/2 =$$

$$\tan \alpha/2 =$$

Ex. 3 Find exact values of:

a) $\cos 15$

b) $\sin (-15)$

Ex. 4 if $\cos \alpha = -3/5$, $\pi < \alpha < 3\pi/2$ Find exact value of:

a) $\sin \alpha/2$ b) $\cos \alpha/2$ 3) $\tan \alpha/2$