

8.1 Polar Coordinates

We have already used the rectangular coordinate system (x,y) where the pt. is a distance from the origin. In a polar coordinate system, we select a pt. called the **pole** and then a ray with vertex at the pole called the **polar axis**.

A pt. P in a polar coordinate system is represented by an ordered pair of numbers (r,θ) . θ can be in degrees or radians.

As an example, suppose that the polar coordinates of a pt. P are $(2,\pi/4)$. First draw an angle of $\pi/4$ radians.

Ex. 1 Finding several polar coordinates of a single pt.

- use $(2, \pi/4)$ from the last page; because $\pi/4, 9\pi/4, -7\pi/4$ all have the same terminal side, we also have pts. $(2, 9\pi/4)$ & $(2, -7\pi/4)$.

In using polar coordinates (r, θ) it is possible for the first entry r to be negative. The pt. is on the ray from the pole extending in the opposite direction of the terminal side at a distance $|r|$ from the pole.

Ex. 2 Polar Coordinate (r, θ) , $r < 0$

Consider $(2, \pi/4)$, this same pt. P can be assigned the polar coordinate $(-2, 5\pi/4)$. To locate the pt. $(-2, 5\pi/4)$, we use the ray in the opposite direction of $5\pi/4$ and go out 2 units along the ray to find pt. P

* These examples show a major difference between rectangular & polar coordinates. In rectangular each pt. has exactly one pair of rectangular coordinates. In polar, a pt. can have infinitely many pairs of polar coordinates.

Ex. 3 Plotting points using polar coordinates

Ex. 4 Finding other polar coordinates of a given pt.

Conversion from polar coordinate to rectangular coordinate:

-if P is a pt. with polar coordinate (r, θ) , the rectangular coordinate (x, y) of P are given by:

$$x = r \cos \theta \quad y = r \sin \theta$$

Ex. 5 Find rectangular coordinate of the following polar coordinates

a) $(6, \pi/6)$

b) $(-4, -\pi/4)$

Ex. 6 Find polar coordinates of a pt. whose rectangular coordinates are $(0, 3)$

Ex. 7 Find polar coordinates of the following rectangular coordinates:

a) (2, -2)

b) (-1, $-\sqrt{3}$)

Steps for converting from rectangular to polar coordinates

1. plot (x, y) and find quadrant
2. find r (distance from origin)
3. find θ

remember: $\theta = \tan^{-1}y/x$ (quad I or IV)

$\theta = \pi + \tan^{-1}y/x$ (quad II or III)