8.5 The Dot Product (Scalar Product)

v **w** is defined by: $a_1a_2 + b_1b_2$

Ex. 1 if **v** = 2i - 3j and **w** = 5i + 3j find:

a) **v** **w**

b) **w** **v**

c) **v** **v**

d) ||**w**||

Properties of the dot product

commutative: **u** **v** = **v** **u**
associative: **u** (**v** + **w**) = **uv** + **uw**
v **v** = $||\mathbf{v}||^2$
0 **v** = 0

Theorem:

if u & v are two non-zero vectors, the angle θ, $0 \leq \theta \leq \pi$, between **u** & **v** is determined by:

$\cos \theta = \mathbf{u} \ \mathbf{v} / ||\mathbf{u}|| \ ||\mathbf{v}||$

Ex. 2 find the θ between $\mathbf{u} = 4i - 3j$ & $\mathbf{v} = 2i + 5j$

Two vectors **v** & **w** are said to be parallel if there is a non-zero scalar α so that $\mathbf{v} = \alpha\mathbf{w}$. In this case, the angle θ between **v** & **w** is 0 or π.

Ex. 3 determine if vectors are parallel

v = 3i - j & **w** = 6i - 2j

if the angle θ between two non-zero vectors **v** & **w** is π/2, the vectors are called orthogonal (perpendicular).

Theorem: Two vectors **v** & **w** are orthogonal iff **v** **w** = 0

Ex. 4 determine whether vectors are orthogonal

v = 2i - j & **w** = 3i + 6j

Projection of a vector onto another vector

****Look at fig. 66 & 67 on pg. 634

Theorem: if **v** & **w** are two non-zero vectors, the vector projection of **v** onto **w** is:

$\mathbf{v}_1 =$

The decomposition of **v** into $\mathbf{v}_1$ & $\mathbf{v}_2$ where $\mathbf{v}_1$ is parallel to **w** and $\mathbf{v}_2$ is orthogonal to **w** is:

$\mathbf{v}_1 =$ $\mathbf{v}_2 =$

Ex. 5 Find the vector projection of $\mathbf{v} = i + 3j$ onto $\mathbf{w} = i + j$. Decompose **v** into two vectors $\mathbf{v}_1$ & $\mathbf{v}_2$ where $\mathbf{v}_1$ is parallel to **w** and $\mathbf{v}_2$ is orthogonal to **w**.