8.7 Cross Product (Vector Product)

For vectors in space only, a second product of 2 vectors is defined, called the cross product. This has applications in geometry and physics.

If $\mathbf{v} = a_1\,\mathbf{i} + b_1\,\mathbf{j} + c_1\,\mathbf{k}$ & $\mathbf{w} = a_2\,\mathbf{i} + b_2\,\mathbf{j} + c_2\,\mathbf{k}$ are 2 vectors in space, the cross product $\mathbf{v} \times \mathbf{w}$ is defined:

Ex. 1 Find cross product of $\mathbf{v} \times \mathbf{w}$ if
$\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ & $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 3\,\mathbf{k}$

Determinants may be used as an aid in computing cross products.

2×2 determinate:

3×3 determinate:

Now ex. 1 using 3 x 3 determinate

Ex. 2 if **v** = 2**i** + 3**j** +5**k** & **w** = **i** + 2**j** +3**k** find:

a) **w** x **v** b) **v** x **v**

Properties of cross product:

Find the Area of a Parallelogram:

Base x Height = $\|\mathbf{u}\| \, [\|\mathbf{v}\| \sin \theta] = \|\mathbf{u} \times \mathbf{v}\|$

Ex. 3 Find area of a parallelogram whose vertices are:
$P_1 = (0,0,0)$, $P_2 = (3,-2,1)$, $P_3 = (-1,3,-1)$, $P_4 = (2,1,0)$

* 2 adjacent sides of this parallelogram are:
$\mathbf{u} = P_1P_2$ & $\mathbf{v} = P_1P_3$